Spin-Hall Conductivity and Pauli Susceptibility in the Presence of Electron-Electron Interactions

Ol'ga V. Dimitrova
L. D. Landau Institute for Theoretical Physics, Russian Academy of Sciences, Kosygina str. 2, Moscow 119334, Russia
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We found the universal relationship between frequency-dependent spin-Hall conductivity and magnetic susceptibility in clean 2D electron systems with Rashba coupling: $\sigma_{sH}(\Omega) = \frac{e}{(g\mu_B)^2 m_b}\chi_\parallel(\Omega)$ in the presence of an arbitrary two-particle spin-conserving interaction. We show that the Coulomb interaction renormalizes the spin-Hall constant. The magnitude of the relative correction to σ_{sH} is proportional to the Coulomb interaction parameter $e^2/\epsilon v_F \hbar$ and does not depend on the strength of the Rashba coupling α.

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Recently it has been proposed[1] that a *dissipationless* spin current can be generated in response to an electric field in semiconductors with the spin-orbital interaction. For the case of an ideal two-dimensional (2D) electron gas with the Rashba coupling, Sinova et al.[2] have found a spin-Hall current of the transverse (z) spin component as a response to an in-plane electric field E_ν, $j^z_\mu = \sigma_{sH}\epsilon_{\mu\nu}E_\nu$, with the "universal" spin-Hall conductivity

$$\sigma_{sH} = \frac{e}{8\pi\hbar} \tag{1}$$

independent of the Rashba interaction constant α and density n, provided that both spin-split bands are occupied. This is the case when the density $n > n^* = m^2\alpha^2/\pi$.

It is important to note that spin current is invariant with respect to time inversion and thus may exist under equilibrium conditions, even without any lateral electric field[3]. For the same reason spin-Hall conductivity belongs to the family of Fermi-liquid response functions, defined generally with respect to space-time-inhomogeneous external electric, $\vec{E}(\Omega,\vec{q})$, and magnetic, $\vec{H}(\Omega,\vec{q})$, fields. A Fermi liquid with Rashba spin-orbital coupling is characterized by two different spin susceptibilities, $\chi_{zz}(\Omega,\vec{q})$ and $\chi_\parallel(\Omega,\vec{q})$, as well as by the lateral dielectric permeability $\epsilon(\Omega,\vec{q})$. Recently, E. Rashba demonstrated[4] a direct relation between the spin-Hall conductivity and the dielectric response function $\epsilon(\Omega,0)$ of a non-interacting 2DEG with spin-orbital interaction. In this Letter we show that the uniform ($q=0$) spin-Hall conductivity is closely related to the in-plane magnetic susceptibility $\chi_\parallel$ as well, providing additional arguments in favor of the equilibrium nature of the spin-Hall constant in a clean 2DEG.

We derive, for a clean (no disorder) 2DEG with an *arbitrary spin-independent electron-electron interaction*, the universal relation between frequency - dependent spin-Hall conductivity and Pauli spin susceptibility of 2D electrons with respect to a spatially uniform parallel magnetic field:

$$\sigma_{sH}(\Omega) = \frac{e}{(g\mu_B)^2 m_b}\chi_\parallel(\Omega), \tag{2}$$

where m_b is the band mass, μ_B is the Bohr magneton and g is the Lande factor.

The relation (2) is valid at any frequency and for any electron density n consistent with the use of a parabolic band spectrum, $\epsilon(p) = p^2/2m_b$. This relation (2) holds even in the case of very low $n < n^*$ when only one chiral subband is populated and the result of Sinova et al.[2], Eq. (1), is not applicable.

Next, we calculate corrections to the spin-Hall conductivity from two-particle electron-electron interactions, and find these corrections to be nonzero. A direct microscopic calculation to the first order in interaction shows that the electron-electron interaction renormalizes both spin-Hall conductivity and in-plane spin susceptibility, while keeping relation (2) intact. The relative magnitudes of these corrections are proportional to the dimensionless Coulomb strength $\frac{e^2}{\epsilon\hbar v_F}$ and do not contain the spin-orbital subband splitting Δ.

Below we provide a brief derivation of the stated results. We start from the formulation of the model of a two-dimensional electron gas with Rashba coupling, which is due to the breakdown of inversion ("up-down") symmetry, leading to an electric field perpendicular to the electron gas plane. It has no effect on the orbital electron motion but it does couple to the electron spin via a relativistic spin-orbit interaction known as the Rashba term[5]. The Hamiltonian of an electron consists of the kinetic energy term and the Rashba term:

$$\hat{h}_{\alpha\beta}(\vec{p}) = \frac{p^2}{2m_b}\delta_{\alpha\beta} + \alpha\left(\sigma^x_{\alpha\beta}\hat{p}_y - \sigma^y_{\alpha\beta}\hat{p}_x\right), \tag{3}$$

where $\hat{p}_\mu = -i\hbar\partial_\mu$ is the momentum of the electron, α is the Rashba velocity, σ^i ($i = x,y,z$) are the Pauli matrices and α,β are the spin indices. Essential to the following discussion is the parabolic band spectrum: $E(p) \propto p^2$. The Hamiltonian (3) can be diagonalized by the unitary matrix:

$$U(\vec{p}) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ ie^{i\varphi_{\mathbf{p}}} & -ie^{i\varphi_{\mathbf{p}}} \end{pmatrix}, \tag{4}$$

where $\varphi_{\mathbf{p}}$ is the angle between the momentum $\vec{p}$ of the

electron and the x-axis, giving the eigenvalues

$$E_\lambda(p) = \frac{p^2}{2m_b} - \lambda\alpha p. \quad (5)$$

The eigenvalues of the chirality operator, $\lambda = \pm 1$, and the momentum of the electron $\vec{p}$ constitute the quantum numbers of an electron state $(\vec{p}, \lambda)$. The Rashba gas has two Fermi circles with the different radii: $p_F = \sqrt{2m_b\mu + m_b^2\alpha^2} \pm m_b\alpha$, where μ is the chemical potential. We assume that spin-orbital coupling is weak, $\alpha \ll v_F = p_F/m_b$. The spin-orbital splitting is then $\Delta = 2\alpha p_F$. The density of states on the two Fermi circles differs as $\nu_\pm = \nu(1 \pm \alpha/v_F)$, where $\nu = m_b/2\pi\hbar^2$. In the following we use units with $\hbar = 1$.

We consider the 2D interacting Rashba electron gas at zero temperature with the Hamiltonian:

$$\hat{H} = \int \psi_\alpha^+(\vec{r})\hat{h}_{\alpha\beta}(\vec{p})\psi_\beta(\vec{r})\ d^2\vec{r} \quad (6)$$
$$+ \frac{1}{2}\int\int \psi_\alpha^\dagger(\vec{r})\psi_\beta^\dagger(\vec{r}')U(|\vec{r}-\vec{r}'|)\psi_\beta(\vec{r}')\psi_\alpha(\vec{r})\ d^2\vec{r}d^2\vec{r}',$$

where $U(|\vec{r}|)$ is an arbitrary two-electron interaction potential; $\hat{h}_{\alpha\beta}(\vec{p})$ is defined in Eq. (3) and $\psi_\alpha^\dagger(\vec{p})$, $\psi_\beta(\vec{p})$ are the electron creation and annihilation operators respectively. Hamiltonian (6) is a rather accurate approximation for the clean two-dimensional semiconducting heterostructures.

The electromagnetic vector potential $\vec{A}$ couples to the orbital motion of the electron according to the transformation: $\vec{p} \to \vec{p} - e\vec{A}/c$, in the Hamiltonian (6). Variation of the Hamiltonian (6) with respect to $\vec{A}$ gives the electric current operator $\hat{J}_\nu = e\int \psi_\alpha^+(\vec{r})(\hat{j}_\nu)_{\alpha\beta}\psi_\beta(\vec{r})d^2\vec{r}$, where the one-particle current operator is:

$$(\hat{j}_\nu)_{\alpha\beta} = e\left(\frac{p_\nu - \frac{e}{c}A_\nu}{m_b}\delta_{\alpha\beta} - \epsilon^{\nu i z}\alpha\sigma^i_{\alpha\beta}\right), \quad (7)$$

with $\nu = x, y$ being the spatial index and $\epsilon^{zi\mu}$ the 3D totally antisymmetric tensor. It is actually a velocity $\hat{j}_\nu = e\hat{v}_\nu$.

Under the non-uniform SU(2) electron spinor transformation, $\psi_\alpha(\vec{r}) \mapsto U_{\alpha\beta}(\vec{r})\psi_\beta(\vec{r})$, the Hamiltonian (6) becomes dependent on the SU(2) "spin electromagnetic" vector potential $\hat{A}_\mu = A^0_\mu\sigma^0 + A^i_\mu\sigma^i$, where A^0_μ coincides with the physical electromagnetic potential and $A^i_\mu = -i\,\mathrm{Tr}(\sigma^i U^+\partial_\mu U)/2$. Although this latter potential is a pure gauge and has no physical consequences, variation of the Hamiltonian (6) with respect to it defines the spin current of the i-th component of spin $\frac{1}{2}$ along the direction μ. The spin current $\hat{J}^i_\mu = \frac{1}{2}\int \psi_\alpha^+(\vec{r})(\hat{j}^i_\mu)_{\alpha\beta}\psi_\beta(\vec{r})d^2\vec{r}$, where the single-particle spin current operator reads as

$$(\hat{j}^i_\mu)_{\alpha\beta} = \frac{1}{2}\left[\frac{p_\mu - \frac{e}{c}A_\mu}{m_b}\sigma^i_{\alpha\beta} + \alpha\epsilon^{i\mu z}\delta_{\alpha\beta}\right]. \quad (8)$$

Our definition of the spin current (8) is equivalent to $\hat{J}^i_\mu = (\hat{v}_\mu\sigma^i + \sigma^i\hat{v}_\mu)/4$, cf. Refs. 1,2,6,7,12,13,15,16,17,18.

To derive the relation (2) between the spin-Hall constant and Pauli susceptibility, we start from two exact commutation relations for total current and spin operators. For the assumed parabolic band spectrum (3), a certain linear combination of the total charge current $\vec{J}$ and the total spin $\vec{S}$ is proportional to the total momentum of the system and commutes with the interaction part of the Hamiltonian. This fact provides us with two exact commutation relations in the presence of an arbitrary spin-conserving two-particle interaction $U(|\vec{r}-\vec{r}'|)$ in the Hamiltonian (6):

$$[\hat{H}, \hat{J}_\mu] = -4iem_b\alpha^2\epsilon^{\mu\nu}\hat{J}^z_\nu$$
and
$$[\hat{H}, \hat{S}^\nu] = 2im_b\alpha\hat{J}^z_\nu, \quad (9)$$

where $\hat{S}^i = \frac{1}{2}\int \psi_\alpha^+(\vec{r})\hat{\sigma}^i\psi_\beta(\vec{r})d^2\vec{r}$ is the total spin of the electron system.

The average spin current of the electron system as a response to weak ac electric field, $E_x(t) = E_{0x}\cos\Omega t$, is given by the general quantum mechanical expression in the first order of perturbation theory[11]:

$$\langle \hat{J}^z_y(t)\rangle = \frac{i}{2}\sum_m \Big[\left(\hat{J}_x\right)_{m0}\Big\{\frac{e^{-i\Omega t}}{\Omega(\omega_{m0}-\Omega-i0)} \quad (10)$$
$$-\frac{e^{i\Omega t}}{\Omega(\omega_{m0}+\Omega-i0)}\Big\}\left(\hat{J}^z_y\right)_{0m} - h.c.\Big]E_{0x},$$

where $\omega_{m0} = \epsilon_m - \epsilon_0$, with 0 being the ground state, and m being the *exact* excitation levels of the interacting system. Note that we have used the Kubo formula (10) for external fields homogeneous in space.

Using the exact commutation relations (9), we can express the matrix elements of the total charge and spin current operators in the right hand side of Eq.(10) in terms of the matrix elements of the total spin operator:

$$\langle \hat{J}^z_y(t)\rangle = -\frac{e}{2m}\sum_m \Big[\left(\hat{S}^y\right)_{m0}\Big\{\frac{e^{-i\Omega t}}{\omega_{mn}-\Omega-i0} \quad (11)$$
$$+\frac{e^{i\Omega t}}{\omega_{mn}+\Omega-i0}\Big\}\left(\hat{S}^y\right)_{0m} + h.c.\Big]E_{0x}.$$

Note that now the right hand side of Eq. (11) is fully analogous (up to a replacement of the e/m_b factor by $(g\mu_b)^2$) to the linear-response expression for Pauli spin susceptibility with respect to an in-plane magnetic field $H_y(t) = H_{0y}\cos\Omega t$, which would replace the electric field E_{0x}. This observation leads us immediately to the relation (2) which is the main result of the present Letter. This relation holds, remarkably, for linear response to perturbations with an arbitrary frequency Ω which are uniform in space, i.e. $q = 0$.

The Fermi liquid response function usually depends on the ratio ω/qv_F. For example, in a normal isotropic Fermi liquid $\chi = 0$ if the limit $q \to 0$, $\omega \to 0$ is taken with the ratio $qv_F/\omega \to 0$ as a consequence of the total spin conservation. The standard Pauli susceptibility $\chi_{Pauli} =$

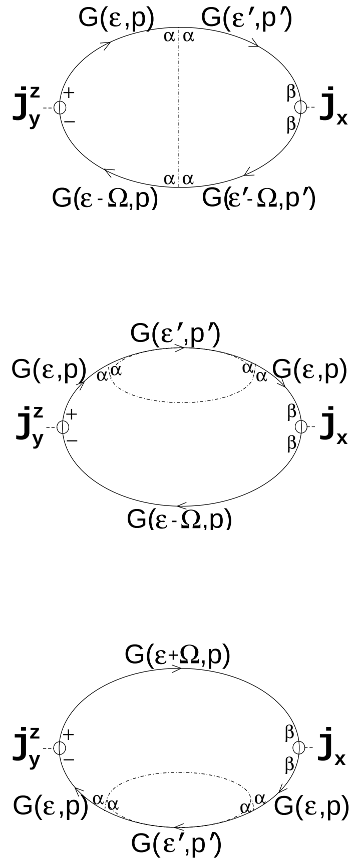

FIG. 1: The correction to the spin-Hall conductivity from electron-electron interaction is given by the sum of the three diagrams, which have equal sign and coefficient. Indices $+,-,\alpha,\beta$ correspond to Keldysh space. Dashed lines correspond to interaction $U(|\vec{p}-\vec{p}'|)$.

$2\mu_B^2\nu(\epsilon_F)$ is obtained with the opposite order of limits, $\omega/qv_F\to 0$ and $q\to 0$. In the case of the Rashba Fermi gas Gor'kov and Rashba[8] have found that $\chi_{zz}=\chi_{\|}=\chi_{Pauli}$ at $\omega/qv_F=0$ and $q\to 0$. We find $\chi_{\|}=\frac{1}{2}\chi_{Pauli}$ and $\chi_{zz}=\chi_{Pauli}$ at $qv_F/\omega=0$ and $\omega\to 0$. Therefore we expect the relation (2) to be valid for $qv_F/\omega\ll 1$.

Next we switch to the second subject of this Letter, which is a calculation of interaction corrections to the spin-Hall conductivity (1). We use the Keldysh technique[9]. In the lowest order of the e-e interaction $U(\vec{r})$, three diagrams, shown in Fig.1, contribute to $\delta\sigma_{sH}$.

Our result is given by Eq. (14).

The averaged Keldysh Green's function is a four by four matrix $\mathcal{G}(p,\epsilon)$ that can be conveniently factorized into a two by two Keldysh matrix whose elements are matrices in spin space:

$$\begin{pmatrix}\mathcal{G}_{--} & \mathcal{G}_{-+}\\ \mathcal{G}_{+-} & \mathcal{G}_{++}\end{pmatrix}=\begin{pmatrix}1-N(p) & -N(p)\\ 1-N(p) & -N(p)\end{pmatrix}G^R(p,\epsilon)+$$
$$+\begin{pmatrix}N(p) & N(p)\\ -1+N(p) & -1+N(p)\end{pmatrix}G^A(p,\epsilon), \qquad (12)$$

where the electron distribution function $N(p)$ is a matrix in spin space. The retarded and advanced averaged Green's functions are diagonal in the chiral basis: $G^{(R,A)}_{\lambda'\lambda}(\epsilon,\vec{p})=G^{(R,A)}_{\lambda}(\epsilon,\vec{p})\delta_{\lambda'\lambda}$, and the solution to the Dyson equation reads[10]:

$$G^{R,A}_{\lambda}(\epsilon,\vec{p})=\frac{1}{\epsilon-\epsilon_\lambda(\vec{p})+\mu\pm i0}\delta_{\lambda'\lambda}. \qquad (13)$$

We choose the gauge for the uniform electric field $\vec{E}(t)=\vec{E}(\Omega)e^{-i\Omega t}$ to be a time dependent vector potential $\vec{A}(t)=\vec{A}(\Omega)e^{-i\Omega t}$, where $\vec{A}(\Omega)=-ic\vec{E}(\Omega)/\Omega$. Using the Keldysh technique we average the spin current operator over the electron state perturbed by both the electromagnetic Hamiltonian, $\hat{H}_{em}=-\frac{1}{c}\int d^2\vec{r}\hat{j}_\nu(\vec{r})A_\nu(t)$, and the electron-electron interaction Hamiltonian, $\frac{1}{2}\int\int\psi^\dagger_\alpha(\vec{r})\psi^\dagger_\beta(\vec{r}')U(|\vec{r}-\vec{r}'|)\psi_\beta(\vec{r}')\psi_\alpha(\vec{r})\ d^2\vec{r}d^2\vec{r}'$, to the first order of perturbation theory. The correction to the spin-Hall conductivity $\delta\sigma_{sH}$ is then found from the relationship $\langle\hat{j}^z_\mu(\Omega)\rangle=\epsilon_{\mu\nu}(\sigma_{sH}(\Omega)+\delta\sigma_{sH}(\Omega))E_\nu(\Omega)$. The resulting expression for the correction to the spin-Hall conductivity from electron-electron interactions reads as follows:

$$\delta\sigma_{sH}=\frac{e}{2V\Omega}\sum_{p,p'}\int\frac{d\epsilon}{2\pi}\frac{d\epsilon'}{2\pi}\mathrm{Tr}\left[A+B+C\right]_{-+}U(|\vec{p}-\vec{p}'|),$$

where

$$\begin{aligned}A&=J^z_y\mathcal{G}(\epsilon-\Omega,p)\left[\tau_z,\mathcal{G}(\epsilon'-\Omega,p')\tau_zJ_x\mathcal{G}(\epsilon',p')\right]_+\mathcal{G}(\epsilon,p),\\ B&=J^z_y\mathcal{G}(\epsilon-\Omega,p)\tau_zJ_x\mathcal{G}(\epsilon,p)\left[\tau_z,\mathcal{G}(\epsilon',p')\right]_+\mathcal{G}(\epsilon,p),\\ C&=J^z_y\mathcal{G}(\epsilon,p)\left[\tau_z,\mathcal{G}(\epsilon',p')\right]_+\mathcal{G}(\epsilon,p)\tau_zJ_x\mathcal{G}(\epsilon+\Omega,p),\end{aligned} \qquad (14)$$

τ^z is the four by four matrix given by the direct product of the Pauli matrix σ^z in the Keldysh space and the unit matrix in the spin space. The current operators in Eq. (14) are the direct products of matrices (7, 8) and the unitary matrix in the Keldysh space. The Tr in Eq. (14) operates only in the spin space whereas the indices $-+$ correspond to the Keldysh space.

Taking the $-+$ element in Keldysh space in Eq. (14), using (12), then taking the trace in spin space and performing integration over the energies, we obtain the expression for the correction to the spin-Hall conductivity in the lowest order of the electron-electron interaction:

$$\delta\sigma_{sH}=$$
$$-e\int\int\frac{d^2\vec{p}}{(2\pi)^2}\frac{d^2\vec{p}'}{(2\pi)^2}\delta N(p)\delta N(p')\,U(|\vec{p}-\vec{p}'|)F(\vec{p},\vec{p}').$$

Here

$\delta N(p)=N_+(p)-N_-(p)$ and

$$F(\vec{p},\vec{p}')=\frac{\cos(\varphi-\varphi')\left\{-p^2+pp'\cos(\varphi-\varphi')\right\}}{16m_b\alpha^2p^2p'^2}, \qquad (15)$$

with $N_\pm(p)$ being the distribution functions of the two Fermi circles of different chiralities. For zero temperature $N_\pm(p)=\theta(-p+p_{F\pm})$.

Explicit integration over momenta in the expression (15) was performed for small spin-orbit interaction $\alpha/v_F \ll 1$ in two limiting cases: short-ranged two-particle interaction (Coulomb potential screened on the lengthscale κ^{-1} smaller than interparticle distance), and full long-range Coulomb interaction. In the Fourrier space these interaction potentials are: $U_1(|\vec{p}-\vec{p}'|) = \frac{2\pi e^2}{\kappa\epsilon}$, and $U_2(|\vec{p}-\vec{p}'|) = \frac{2\pi e^2}{\epsilon|\vec{p}-\vec{p}'|}$. The final expressions for σ_{sH} in these two cases are as follows:

$$\sigma_{sH}^{(\mathrm{short})} = \frac{e}{8\pi\hbar}\left[1 - \frac{m_b e^2}{2\epsilon\kappa}\right] \tag{16}$$

for the short-range potential and

$$\sigma_{sH}^{(\mathrm{Coulomb})} = \frac{e}{8\pi\hbar}\left[1 - \frac{2m_b e^2}{3\pi\epsilon p_F}\right] \tag{17}$$

for the Coulomb potential. It is seen that the correction to the spin-Hall conductivity is independent of the spin-orbit constant α (in Eq. (17) corrections of the order $(\alpha/v_F)^2 \ll 1$ are neglected), and is proportional to the standard Coulomb interaction parameter $e^2/\epsilon\hbar v_F$.

For completeness we have performed direct diagram calculations of the interaction correction to the in-plane susceptibility, represented by three diagrams similar to those shown in Fig.1. The results for the relative corrections to the in-plane susceptibility were found to coincide with expressions (16,17), in agreement with the general relation (2).

We checked by direct calculation for a clean system without interaction that spin susceptibilities and spin-Hall conductivities, Eq. (18), for systems of fermions with higher spins j follow relation (2). Interestingly we find[19] that in the case of an ideal 2D Rashba gas of fermions of arbitrary half-integer spin j the value of the spin-Hall constant is also universal and grows with j:

$$\sigma_{sH}(j) = \frac{e}{4\pi}\sum_{m=-j}^{j} m^2. \tag{18}$$

In this Letter we did not discuss the very actively debated issue of the stability of the spin-Hall response to disorder, with quite a few of conflicting results presented during last months[12,13,14,15,16,17,18]. We expect our results to be directly relevant for submicron samples of a very clean electron gas, with the sample size less than the elastic scattering length. The influence of disorder upon σ_{sH} in the presense of electron-electron interactions is to be studied separately.

In conclusion, we have shown that the frequency-dependent spin-Hall conductivity and Pauli susceptibility of a clean interacting 2D Rashba EG are proportional to each other, with the coefficient containing band mass, Lande factor and Bohr magneton only. We calculated the first-order interaction-induced correction to the spin-Hall conductivity and found it to be proportional to the standard dimensionless interaction strength. At the final stage of preparation of this Letter, we became aware of the paper[20], where a similar relation between spin-Hall conductivity and Pauli susceptibility is discussed for a non-interacting Rashba electron gas.

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